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Mixed convection from long horizontal cylinders

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This paper is the report of an experimental study of heat transfer from fine horizontal wires of various lengths to fluids having various Prandtl numbers. Transport characteristics were determined for the full spectrum of processes from natural, through mixed, to forced convection. The experimental results are compared with those of past analysis and with correlations. The very high level of accuracy and reproducibility of the data makes possible the detection of very small effects. The influence of cylinder length on transport is shown for both of the asymptotic régimes of natural and of forced convection. Transport in the mixed-convection régime is determined and limits of that régime are estimated.

1. Introduction

The heat-transfer characteristics from long horizontal circular cylinders and wires to fluids are very important in many applications in technology and are a good comparative index for flows over other cylindrical shapes. Physical circumstances of interest include many different conditions and characteristics of flow, of property variation, fluids, etc. Transport in both the natural and the forced convection régimes is important and there are a number of different flow subrégimes in each. In many circumstances the actual flow is a combination of the characteristics of these two extreme modes and is called mixed convection. The boundaries between the three régimes are also important.

In addition, for any one of the régimes, there is a question concerning the dependence of transport characteristics on the length L of cylinder, compared to its diameter D. A particular question concerns the required magnitude of L/D so that this quantity no longer remains an important variable. That is, how large must L/D be in each of the various régimes so that the transport is equivalent to that of a cylinder of infinite length.

Another aspect of principal importance is the nature of the dependence of transport on the Prandtl number of the fluid. In all régimes this parameter indicates the relative extent of the thermal and velocity fields which control transport. This parameter is important in understanding experimental results and in fashioning appropriate techniques of analysis to these difficult and complicated flows.

There are also applications in which non-continuum, compressibility and/or viscous dissipation effects are important. There have also been studies for fluids near their critical state. However, these latter questions are not considered further here.

The present paper summarizes the results of an experimental study of the transport characteristics in the natural, forced and mixed convection régimes for very small diameter electrically heated cylinders (wires) of various lengths in fluids having Prandtl numbers in the range 0·70–63. Flow was upward. Thus, the two effects were always parallel and in the same direction. The experimental data, temperatures, heat-transfer rates, and velocities were determined at an uncommonly high level of repeatibility, and therefore, permit the detection of very small differences and changes in the nature of transport behaviour. These results are compared with commonly accepted methods of correlation for pure natural convection and with correlations and analysis for pure forced convection. The data and predictions of the L/D effect are very interesting as are the limits of the mixed-convection régime.

Before presenting the new experimental results, several aspects of the transport circumstance will be reviewed and related to what is already known. Since the experimental study concerned small diameter wires, for which the size-based parameters of Reynolds number (Re) and Grashof number (Gr) are very small, the discussion will be limited to information relevant to low Reynolds- and Grashof-number flows. Note that $Gr = g\beta D^3(t_0 - t_\infty)/\nu^2$ and at $Re = U_\infty D/\nu$, where $(t_0 - t_\infty)$ is the temperature difference between the wire and the fluid and β is the volumetric coefficient of thermal expansion.

There have been many analytical studies of forced flow over cylinders assumed to be of infinite length. The studies of concern here are those for low Reynolds number. Early solutions for both flow and heat transfer employed Oseen approximations. In the more recent studies of flow by Kaplun (1957) and Proudman & Pearson (1957) and of heat transfer by Hieber & Gebhart (1968) for moderate and large Prandtl numbers, the method of matched asymptotic expansion has been used. It is thought that this method of solution may be expected, in successively higher approximations, to yield results of very high accuracy.

The analysis of natural convection is very much more complicated. The method of matched asymptotic expansion is not simply applicable in this problem. The writers are not aware of any but very approximate solutions for natural convection over a long horizontal cylinder in a uniform ambient medium. Similarly we know of no published analytical studies of mixed convection for the cylinder geometry.

There have been many experimental studies of both forced convection and natural convection over long horizontal cylinders and many suggested correlations of the heat-transfer behaviour. For forced convection the wide Reynolds number correlations of Hilpert and of McAdams, for air, are well known. For natural convection the early correlation by Nusselt considered data over a very wide range of conditions for various common fluids. In recent years, many additional correlations of considerable engineering value have appeared which consider a wide range of Reynolds and Grashof numbers, for various Prandtl-number fluids. However, since the internal spread of data used in these correlations is much greater than that of good analytical results and of current experimental accuracy, these results are not of value here.

However, several studies are of particular interest. Piret, James & Stacy

(1947) considered forced flow of water (nominal Prandtl number of 6·8) over a fine wire, for $Re > 0{\cdot}08$. A correlation curve was derived from the data. These results are of somewhat limited general applicability since large temperature differences, 25–70 °C, caused first-order property variations. Also, the L/D ratio was only 1000. Another study by Collis & Williams (1959) suggests a correlation for air over a Reynolds-number range from 0·01 to 140, including an allowance for temperature-difference effects on properties. The experiments were performed on wires of very large L/D ratio. The correlation is in excellent agreement with the analytical results of Hieber & Gebhart (1968), in the low Reynolds-number range. Collis & Williams propose that natural convection effects are negligible for $Re > Gr^{\frac{1}{3}}$, in air.

For pure natural convection, the study of Collis & Williams (1954), in air, is of particular interest here. The effect of L/D was studied and values of the order of 10^4 were found to be necessary to exclude the effect of L/D. A correlation for heat transfer was presented which progressively deviated from the traditional wide-range correlations below $Gr = O(10^{-2})$. These results will be used in a comparison with our results in air since our values of Grashof number are very small.

The experimental results reported in the present paper span the whole range of processes from pure forced, through mixed, to pure natural convection and the above pure régime results are of importance in validating our experiments in the asymptotic régimes. There has been some previous work in mixed convection for this geometry. Anantanarayanan & Ramachandran (1958), Sreenivasan & Ramachandran (1961), Deaver, Penney & Jefferson (1962), Penny & Jefferson (1966), Mabuchi & Tanaka (1967), and Gupta & Agrawal (1968) report studies of mixed convection over horizontal wires which were vibrated (or oscillated) in a vertical plane at various frequencies and amplitudes in air, water, and ethylene glycol. The L/D values were considerably less than 10^4 and it is though that even the low frequency and large amplitude data is not useful for comparison with our results.

A very recent study by Oosthuizen & Madan (1970) reports experimental results in the mixed-convection range for vertical flow over cylinders of 30 cm length and diameters ranging from approximately 2 to 4 cm, an L/D range from 8 to 15. The resulting Reynolds and Grashof number ranges of 0–3000 and 25,000–300,000, respectively, produced both pure régimes as well as first-order mixed-convection effects, over most of the intervening region. Sharma & Sukhatme (1969) present similar information for horizontal flow of air over cylinders of 1·25–5·05 cm diameter and 30 cm length. However, the Grashof- and Reynolds-number values place these studies outside the range of the results of the present study, which concerns very small diameter and large L/D wires.

The results of Van Der Hegge Zijnen (1956) follow from an attempt to produce a correlating equation for mixed convection for horizontal cylinders by a combination of correlating equations of pure forced and pure natural convection. Experimental results are presented for two cylinders, one of 0·01 cm diameter and having an L/D ratio of 550 and another cylinder having a diameter of 0·904 cm, of unstated length. Although the apparent intent of this work was to determine the conditions under which natural-convection effects become appreciable in a predominately forced-convection process, the results are inconclusive.

The foregoing account of what was known until very recently about mixed convection over cylinders, in the continuum of processes between the pure asymptotic régimes, consists essentially of an indication of the lower limit of pure forced convection in air for fine wires, i.e. for low Reynolds-number flows, and some detailed information concerning mixed-convection transport in the high range of Reynolds and Grashof numbers.

A very recent paper by Gebhart, Audunson & Pera (1970) reports the results of an experimental study of natural, forced and mixed convection for fine horizontal wires in air and in two silicone liquids, $Pr = 6{\cdot}3$ and 63, for a single wire having an L/D of 8000. These experiments were carried out in the new apparatus described by Dring & Gebhart (1969) as subsequently modified to obtain very high accuracy data. This study indicated that the accuracy and reproducibility of the reduced transport data were generally somewhat better than 1 %.

This limited data indicated conclusively that the traditional method of correlating natural-convection heat transfer using the single parameter $GrPr$ leaves a residual Prandtl-number effect of the order of 15 %. It was also shown that the experimental forced-convection correlation of Piret *et al.* (1947) was 23 % higher than the matched asymptotic solution of Hieber & Gebhart (1968) for the Prandtl-number level of water. The data for $L/D = 8000$ was about midway between the two, suggesting that this L/D was too small for effectively infinite length behaviour in this liquid. Data for $Pr = 63$ were also above the solution. However, the forced-convection data in air were in excellent agreement with both the experimental results of Collis & Williams (1959) and the solutions of Hieber & Gebhart (1968) and with the solution of Kassoy (1967), Dennis, Hudson & Smith (1968) and Wood (1968) for Prandtl numbers around 1·0. This suggests that an L/D of 8000 is adequate in gases for forced convection.

The data were also analyzed to determine the limits of the mixed-convection régimes. It was apparent that the parameter $Re/Gr^{\frac{1}{3}}$ would not be generally useful in determining the limit of essentially pure forced convection. The data at the natural-convection side of the mixed-convection régime were difficult to interpret because of the very small velocities and effects and because there was some evidence that external disturbances influenced these data.

The results of this preliminary experimental study indicated many important unanswered questions concerning the total spectrum of transport régimes and their limits, about the effect of Prandtl number on transport, and concerning the nature of the effect of cylinder L/D. Therefore the translating equipment was rebuilt and better isolation was provided. The equipment was enlarged to accommodate L/D values to 16,000. The instrumentation was improved to provide an accuracy and reproducibility level of the order of 1 % in the mixed-convection régime and even better in the pure natural-convection régime for temperature differences greater than 4 °F.

A series of experiments was carried out in natural convection for L/D values from 1000 to 16,000 for $Pr = 0{\cdot}7$, $6{\cdot}3$ and 63. Mixed- and forced-convection experiments were carried out for L/D values of 8000, 12,000 and 16,000 for

$Pr = 6{\cdot}3$ and 63. The air case was not redone here, since the previous results were conclusive.

The results of this study are compared with previous predictions for forced and for natural convection and are used to determine the limits of the mixed-convection régime. Transport behaviour for mixed convection is shown. The L/D effects and limits are indicated.

2. Experimental procedure and accuracy

The principal components of the experimental set-up are shown in figure 1. All tests were run in a 9 in. internal diameter stainless-steel superinsulated double-wall dewar flask. The container could be raised and lowered at controlled and uniform speeds ranging from 0·157 inches/min to 314 inches/min. The micro-switches S_1 and S_2 stop the container at the ends of the travel and the micro-switches S_3 and S_4 operate the 6-digit integrating digital voltmeter (Vidar 520 IDVM) and the timer (Beckman model 5230B). The distance travelled between S_3 and S_4 could be measured to within a fraction of $\frac{1}{1000}$ in. with the micrometer shown.

Wire length (in.)	L/D	$\alpha(1/^{\circ}\mathrm{F})$
0·39850	996	0·001880820
0·60840	1,521	0·001884535
1·09400	2,535	0·001930528
1·62450	4,061	0·001897452
3·23825	8,095	0·001915000
4·79800	11,995	0·001857078
6·48007	16,200	0·001873043

TABLE 1. Measured characteristics of wires used

The wire probe was stationary and consisted of a 0·00040 in. platinum wire suspended between two prongs made of $\frac{1}{16}$ in. diameter spring steel. The prongs were tapered to minimize any disturbance they might cause in the flow. The platinum wire was soft-soldered to the prongs and was drawn tight by moving the prong supports apart with a finely threaded screw. The lengths of the various wires, which were measured in an optical comparator, are given in table 1. The diameter of the wires was measured in a Reichert metallographic microscope, using an eye-piece with a cross-hair which was moved across the field by turning a graduated barrel. The eye-piece was calibrated by placing a grating with a $\frac{1}{1000}$ in. division in front of the objective. For the particular setting used, each division on the eye-piece barrel was found to represent 0·000005 in. The average of the diameter, measured at twenty-one locations along an arbitrary 1 in. sample, was found to be 0·00040 in., with a standard deviation of $\pm 1\ \%$.

Each probe was calibrated for electrical resistance in a constant-temperature oil bath. Over the temperature range 70–119 °F the resistance change with temperature was strictly linear. Values of the resistance–temperature coefficient for the different probes are also given in table 1. The resistance of the platinum wire used is approximately 40·5 Ω/in. at 78 °F. The temperature difference needed to dissipate the calibration current through the wire was calculated of the order of 0·01 °F. Therefore, the temperature of the oil bath was assumed to be equal to the temperature of the wire. This assumption was checked by using calibration or trickle currents varying from 0·6 to 1·1 mA. The effect of this variation upon the measured wire resistance was negligible. All final calibrations were done with a current of about 1 mA.

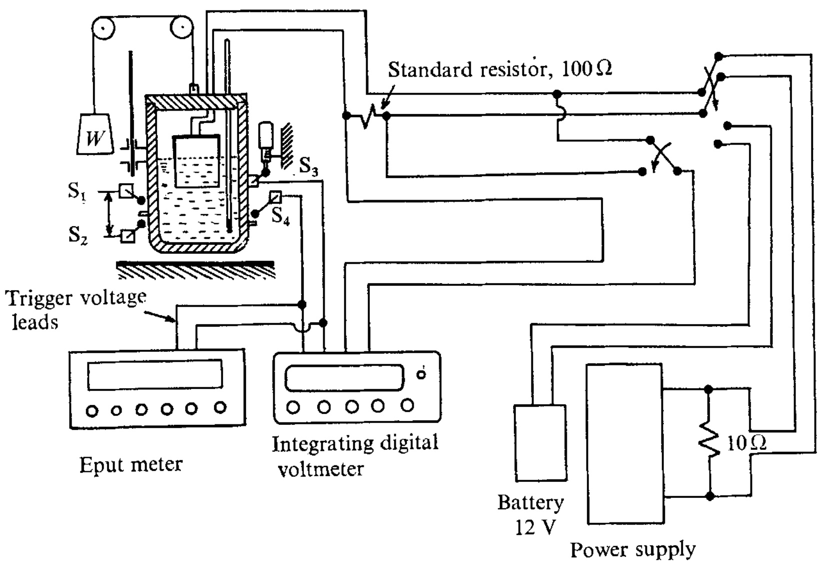

FIGURE 1. Diagram of test apparatus.

The heat-dissipation rate of the wire was calculated from digital voltmeter measurements of the voltage drop across the wire and across a standard resistor in series (for current). The cold resistance of the wire was measured in each test by passing a trickle current through it.

The experimental arrangement did not allow simultaneous recording of both voltage and current. However, the constancy of the regulated power source made it possible to measure the current and voltage of two successive traverses. The small variation in both current and voltage was averaged by integration over the accurately known traverse time interval. For each wire voltage setting, the above measurements were repeated three times. For each tank velocity, data were taken at a sufficient number of wire voltage settings to cover a predefined range of Grashof numbers.

It was important that all data be taken under steady flow conditions. Such conditions were obtained by having the tank travel for approximately one inch before any data were taken. The attainment of steady-state conditions was verified by observing the continuous current reading on the IDVM. These observations showed that steady flow was reached very quickly, as was expected with such a fine wire.

Data collection was started by the engagement of the switch referred to as S_3 in figure 1 and terminated by the closing of switch S_4, i.e. all data were recorded while the tank was rising. Distances travelled by the tank between the switches S_3 and S_4 ranged from 0·3 in. at high speed to 0·05 in. at low speed.

The ambient temperature in the tank, for fluid property evaluation, was measured before and after each run with a mercury in glass thermometer of 0·1 °F/division. In order to ascertain that the fluid was not stratified, the voltage drop across the wire, using the trickle current, was measured with the tank in both its upper and lower positions.

The greatest source of error in the experiment is the measurement of the temperature difference between the wire and the fluid. This error is greatest for small temperature differences. For example, an error of 0·05 °F in the temperature determination of the wire would result in a 1 % error in the corresponding Nusselt number for a 5 °F overall temperature difference. Such an error could be caused by a combination of the uncertainty in the wire calibration (0·1 %), by a small fluid stratification that may appear during the experiment, and on the reading of the mercury-in-glass thermometer. Therefore, our mixed convection results have a larger error in the low Grashof number range where the overall temperature difference is of the order of 4 °F. The error is greatly reduced in the high Grashof number range, where the wire–fluid temperature difference is as high as 45 °F. For pure natural convection, temperature differences down to 1 °F were used, resulting in a somewhat lower level of accuracy.

The other sources of error were found to be negligible. For example, the integrating digital voltmeter used to obtain the data is a high-impedance instrument (1 MΩ) and is accurate to 0·01 % of the full six-digit scale. The electric current was measured across a Leeds and Northrup standard resistor of 100·0008 Ω, of 0·0001 % accuracy in the range of current used. The wires were soft-soldered to the supporting prongs to minimize contact resistance. The radiation losses were computed and found to be less than 0·01 % of the total heat transfer from the wire and therefore were neglected throughout. Calculations indicated that the end losses due to metallic conduction were negligible for such fine and long wires.

Building vibrations and vibrations transmitted by the pulling mechanism were also considered as a source of inaccuracy in the low-velocity range. The apparatus was mounted on absorbing pads and the pulling mechanism was vibration mounted separately on a solid wall, thus reducing these effects to a negligible level.

All fluid properties were evaluated at the mean film temperature (average between distant fluid and wire temperatures). The properties of air were taken from standard tables. The density and viscosity of the samples of the silicone oils SK-96 (0·65) and SK-96 (5) used were especially determined by the manufacturer.

3. The experiments and results

To conduct the experiments, a given wire length was mounted and calibrated. The characteristics of particular wires are listed in table 1. For each of the nominal L/D values of 8000, 12,000 and 16,000 experiments were run for the full range of velocities at various temperature differences, in each of the liquids, $Pr = 6{\cdot}3$ and 63. Air was not used because previous results were thought to be adequate and because the required velocity range was too high for an accuracy level consistent with other results. These three probes, as well as those for nominal L/D values of 1000, 1500, 2500 and 4000, were used in each of three fluids at rest to determine the asymptotic pure natural-convection transport characteristics over the range of Grashof number resulting from temperature differences of approximately 1–45 °F in the silicon fluids and of 2–80 °F in air.

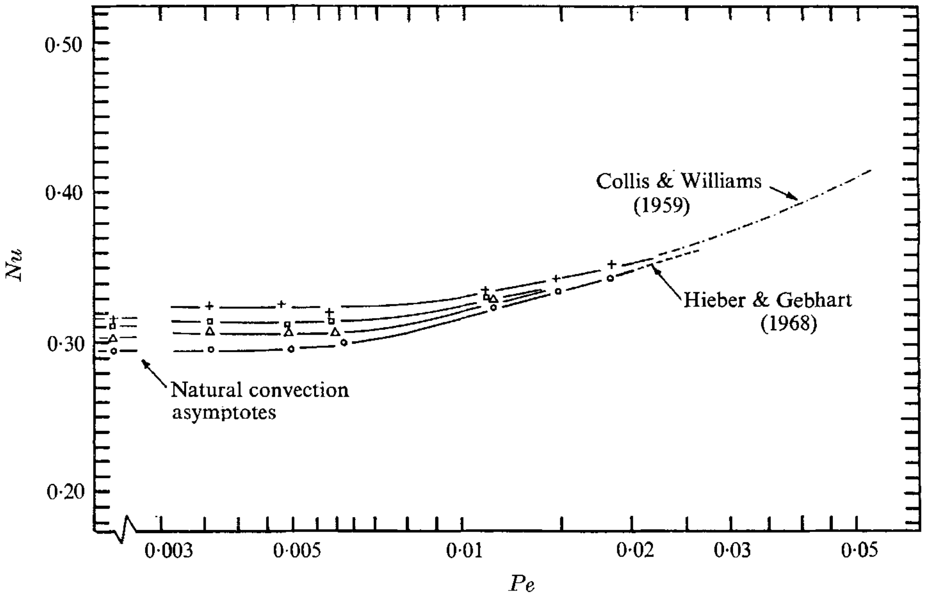

FIGURE 2. Measured combined convection transport in air at atmospheric conditions. $Pr = 0{\cdot}7$, $L/D = 8000$. Ra: $+$, $0{\cdot}21\times10^{-5}$; $\square$, $0{\cdot}14\times10^{-5}$; $\triangle$, $0{\cdot}10\times10^{-5}$; $\bigcirc$, $0{\cdot}07\times10^{-5}$.

A given traverse of the mixed-convection region required the determination of transport data at each point for the same Grashof number, i.e. temperature difference, so that contours of constant Gr could be drawn. This was difficult since the heating rate was controlled, not the temperature difference. Therefore, at each velocity several runs were made, at slightly different heating rates, to permit the inference of transport behaviour at a particular exact value of the Grashof number, or Rayleigh number, $Ra = Gr\,Pr$.

The resulting data for air, from Gebhart *et al.* (1970) is shown in figure 2. The new data for the two liquids is shown in figures 3 and 4. The Nusselt number $Nu = hD/k$ (h is the convection coefficient and k is the thermal conductivity of the fluid) is plotted *vs.* the Péclet number, $Pe = Re\,Pr$. The pure natural-convection asymptotes are shown. Note that the Nusselt number is always greater for smaller L/D for a given condition. This suggests end losses. However, any simple form of end-loss calculation is inadequate to account for these large effects.

Figure 2 shows the excellent agreement of the data in air for $L/D = 8000$ with previous experimental and analytical results for forced convection. The inference

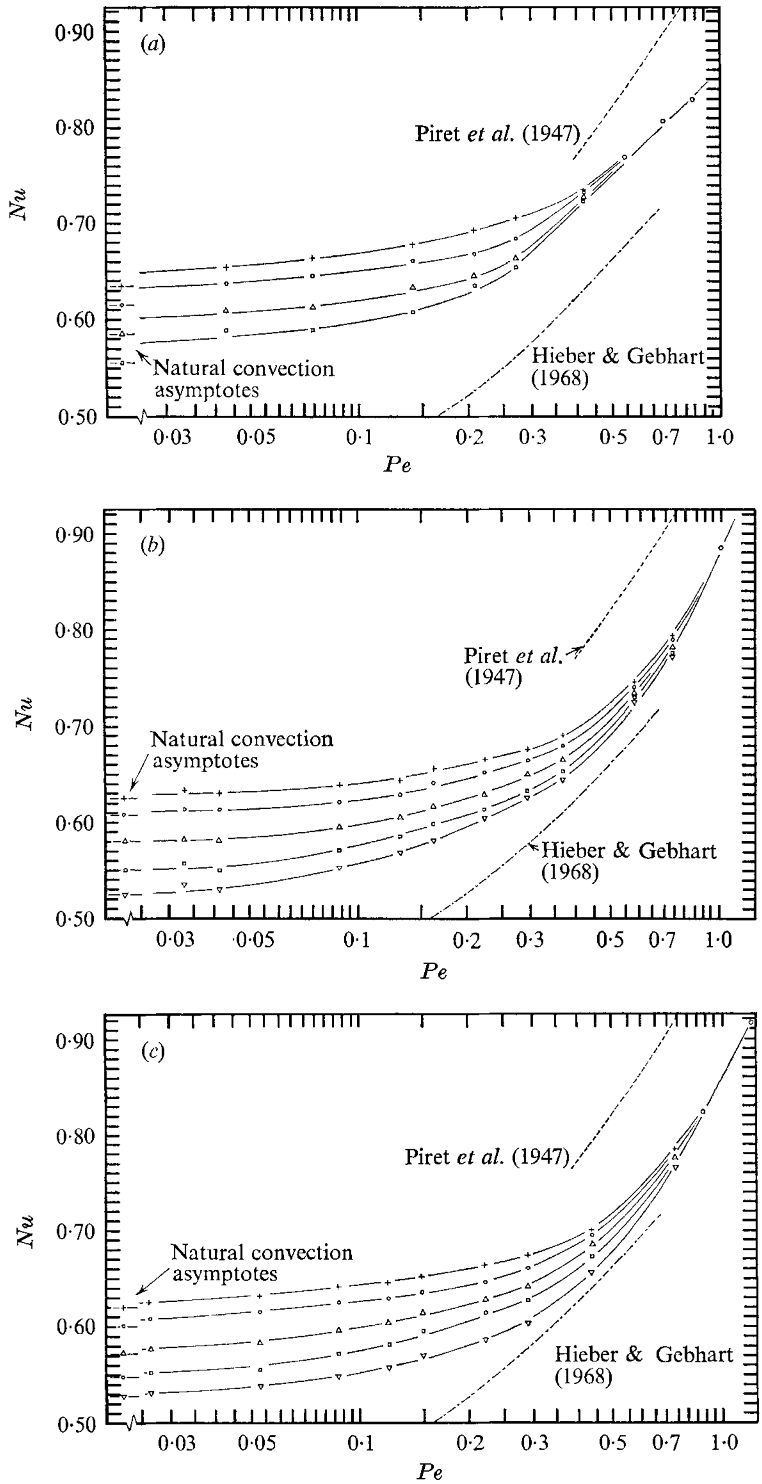

FIGURE 3. Measured combined convection transport in 0·65 centistoke silicone at various length–diameter ratios. (*a*) $Pr = 6{\cdot}3$, $L/D = 8000$. (*b*) $Pr = 6{\cdot}3$, $L/D = 12{,}000$. (*c*) $Pr = 6{\cdot}3$, $L/D = 16{,}000$. *Ra*: $+$, 6×10^{-3}; $\bigcirc$, 4×10^{-3}; $\triangle$, 2×10^{-3}; $\square$, 1×10^{-3}; ∇, $0{\cdot}5\times10^{-3}$.

is that this L/D is adequate for essentially infinite length forced-convection behaviour. However, this is not true in natural convection, as will be shown later.

For the light silicone ($Pr = 6{\cdot}3$) the L/D effect is seen in figure 3 to be appreciable between 8000 and 16,000. The results for 16,000 are 3 % higher at $Pe = 0{\cdot}5$ than the analytical result. The previous correlation of data by Piret *et al.* (1947), at $L/D = 1000$, is seen to be 23 % higher. These results suggest that lower limit of L/D is still somewhat higher than 16,000 for this fluid. The mixed-convection contours are seen to approach the two limits smoothly and the variation of Nusselt number over the whole range is relatively small.

Similar information in figure 4 for the heavier silicone ($Pr = 63$) also indicates an appreciable L/D effect in the range of 8000–16,000. Even for the highest value the experimental results for pure forced convection are 12 % greater than the theory. This is surprising since the results of the theoretical analysis, with a small number of terms evaluated, should become more accurate at higher Prandtl numbers since the temperature disturbance is restricted even more deeply to the pure Stokes region of the flow. We conclude, therefore, that in the more highly viscous fluid the wire wake induces a much more vigorous three-dimensional effect and that much greater L/D values are necessary for effectively infinite length behaviour. Recall that the L/D requirement is greater for $Pr = 6{\cdot}3$ than for air. Again the mixed convection contours are uneventful.

In figure 5 are plotted all of the pure natural-convection régime results. The region where temperature difference is less than 4 °F, where the accuracy of the results is somewhat lower, is indicated in the figure for each fluid. The L/D effect is seen to be small for the liquids and larger in air. The Nusselt number dependence on L/D is plotted in figure 6 for some values of Ra in the high-accuracy ranges.

The data for $L/D = 16{,}000$ is plotted in figure 7 *vs.* Rayleigh number, the conventional procedure. The data for air is 3 % lower than the correlation of Collis & Williams (1954) for effectively infinite-length cylinders.

These natural-convection data do not correlate with the suggestion of Nusselt, nor with any reasonable single curve drawn across this range of five orders of magnitude of the Rayleigh number. Clearly data at various Prandtl numbers does not correlate with Rayleigh number unless one is willing to accept errors of the order of 15 %. Closer examination of data used in past correlations indicates a systematic deviation of data, with Prandtl number, from any one-line correlation.

The other aspect of the data of particular importance is the indication of the limits of the complicated régime of mixed convection for this geometry and flow circumstance. The limits are usually expressed in terms of the condition wherein transport in the pure régime of either natural or forced convection is appreciably changed by the other effect. A change in transport of 5 or 10 % might be used as a criterion.

Order of magnitude arguments with the equations of motion and boundary conditions indicate that the two mechanisms are comparable in effect on transport when the parameters Re and $Gr^{1/n}$ are of the same order, where n is perhaps 2 or 3.

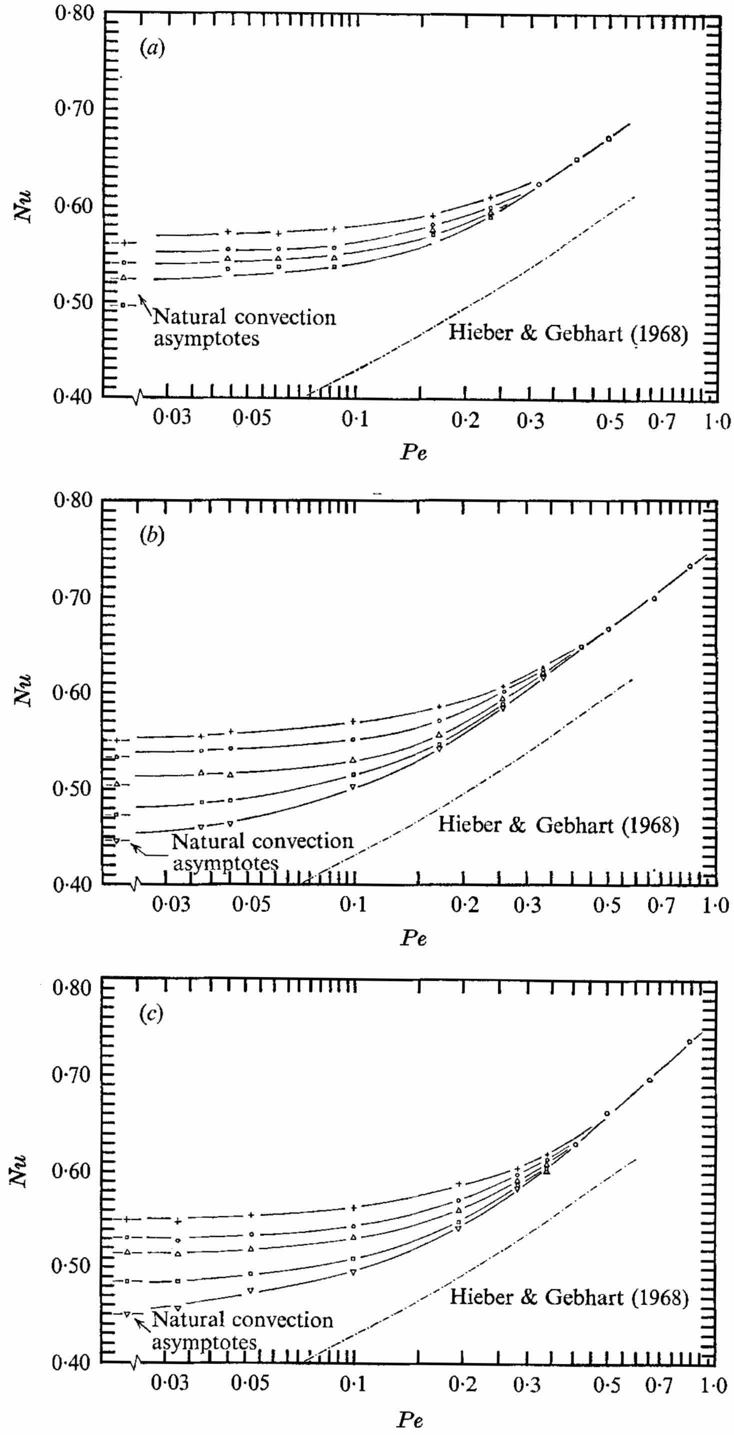

FIGURE 4. Measured combined convection transport in 5·8 centistoke silicone at various length–diameter ratios. (*a*) $Pr = 63$, $L/D = 8000$. (*b*) $Pr = 63$, $L/D = 12{,}000$. (*c*) $Pr = 63$, $L/D = 16{,}000$. (*a*) and (*b*) *Ra*: $+$, 5×10^{-4}; $\bigcirc$, 3×10^{-4}; $\triangle$, 2×10^{-4}; $\square$, 1×10^{-4}; ∇, $0{\cdot}5\times10^{-4}$. (*c*) *Ra*: $+$, 6×10^{-4}; $\bigcirc$, 4×10^{-4}; $\triangle$, $\square$, ∇ as in (*a*) and (*b*).

Therefore it is reasonable to express the two boundaries of mixed convection as

$$Re = CGr^{1/n},$$

where C and n are determined from experimental data such as that in figures 2–4.

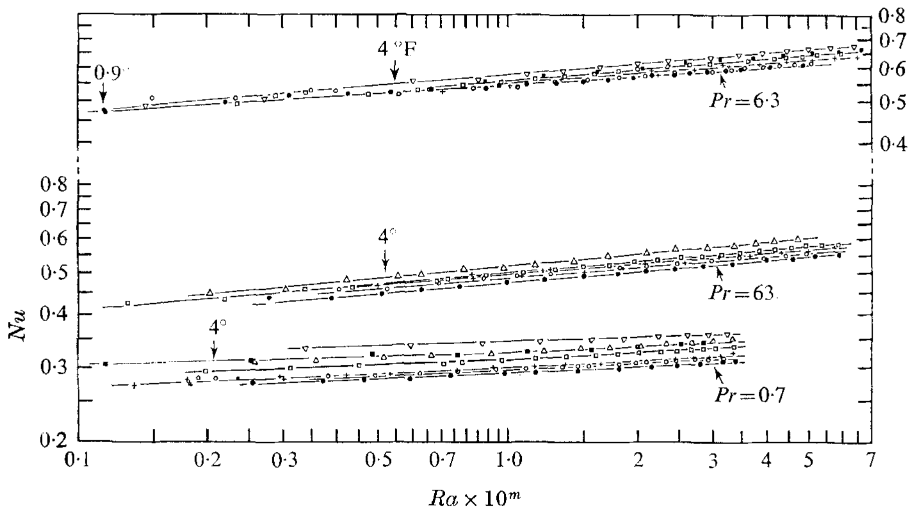

FIGURE 5. Effect of wire length on natural-convection heat transfer for various Prandtl-number fluids. For $Pr = 6{\cdot}3$, $m = 3$; for $Pr = 63$, $m = 4$; for $Pr = 0{\cdot}7$, $m = 6$, where m is the exponent of 10 in the abcissa above. L/D: ▽, 1000; ■, 1500; △, 2500; □, 4000; +, 8000; ○, 12,000; ●, 16,000.

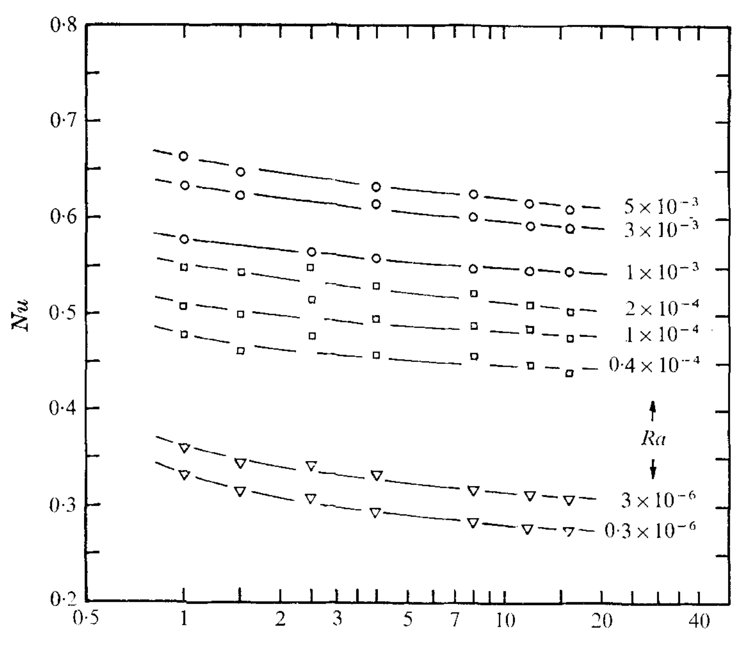

FIGURE 6. Effect of wire length on natural-convection heat transfer. ○—○, $Pr = 6{\cdot}3$; □—□, $Pr = 63$; ▽—▽, $Pr = 0{\cdot}7$.

The necessary-flow Reynolds numbers for various percentage increases in transport over pure natural convection are plotted for various Grashof number levels in figure 8 for the data in liquids and for $L/D = 16{,}000$. The inherently high accuracy of the experimental results may be seen from the orderly progression of these points evaluated with such a small change in effect. The curves drawn in figure 8 indicate the values of C and n for the various limits of the forced-convection effect. There is clearly an L/D effect. However, the Prandtl-number effect on both C and n is seen to be larger than that of L/D. Values of n for the curves shown vary over the range 2·85–3·76 for $Pr = 6{\cdot}3$ and over the range 2·00–2·63 for $Pr = 63$, having average values of 3·34 and 2·30, respectively.

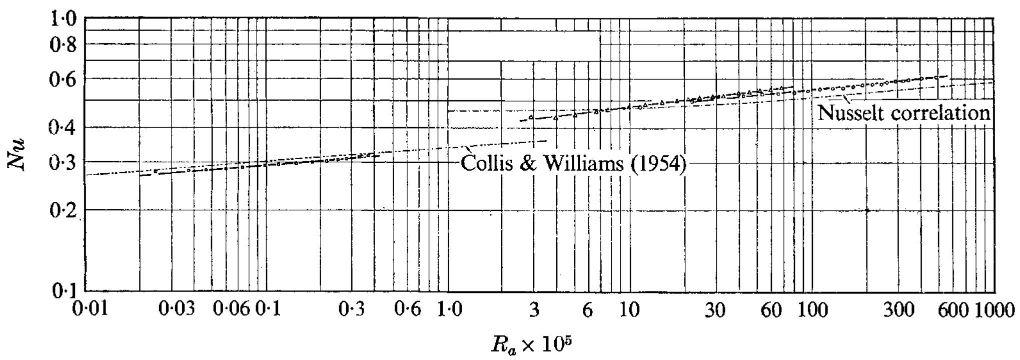

FIGURE 7. Natural-convection heat transfer for fluids of various Prandtl numbers. $L/D = 16{,}000$. Nominal Pr: □, 0·7; △, 63; ○, 6·3.

Similarly, results are seen in figure 9 for the incipient natural-convection effect on transport in forced convection. Data for various L/D ratios are plotted for all three fluids and the curves through the points again indicate the values of C and n required for this limit. Values of n are given in table 2 for each fluid and L/D. These are values averaged over the various percent effect levels.

These results, concerning limits, are certainly not simple or easily generalized. They show, for the first time perhaps, the extreme complexity of the general problem of transport with the interaction of forcing conditions with density-induced buoyancy forces.

Comparison of parts (*a*), (*b*) and (*c*) of figure 4 shows in another way the especially large effect of L/D on mixed convection at high Prandtl number. In addition to the sharp dependence on L/D in the asymptotic régimes, mixed-convection transport is much higher at smaller L/D. Also, the approaches to the relevant asymptotes, especially to those of natural convection, are much slower at decreasing Péclet number at smaller L/D.

4. Conclusions

The data reported here show many interesting characteristics of heat transfer from long cylinders in the various régimes. There is no theory for natural convection transport in the low Grashof-number régime and these results clearly

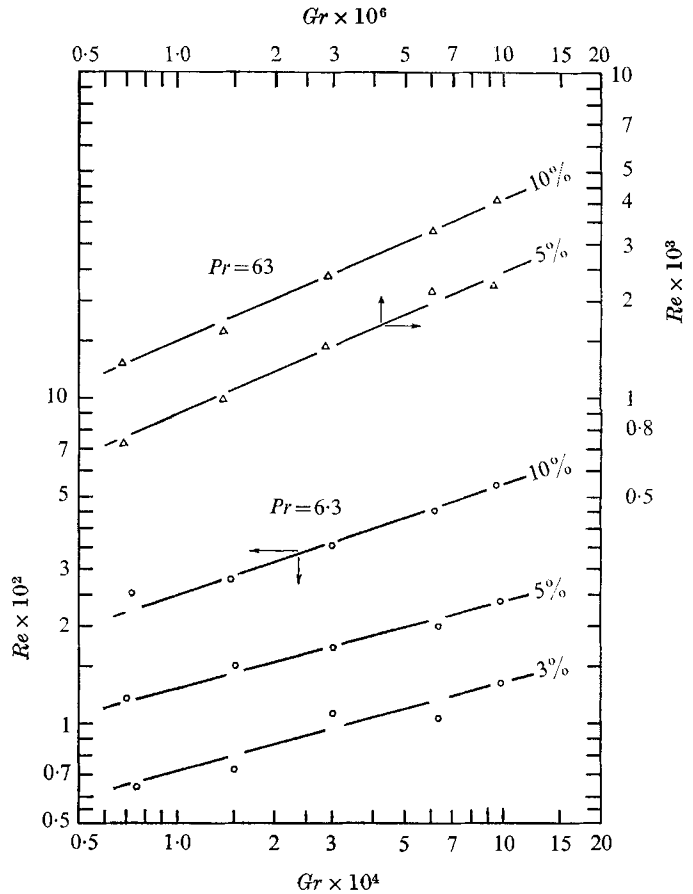

FIGURE 8. Limits of the natural-convection régime. Percentage changes in transport due to forced convection are indicated. $L/D = 16{,}000$.

Pr	$L/D = 8000$	$L/D = 12{,}000$	$L/D = 16{,}000$
0·7	1·97	—	—
6·3	2·78	3·92	3·42
63	3·30	4·16	3·14

TABLE 2. Limits of the effects of natural convection on forced convection

indicate that the conventional technique of correlation in terms of Rayleigh number is not an adequate allowance for the Prandtl-number effect. On the other hand, it is shown that a recent calculation of the Prandtl-number dependence in forced convection accurately predicts this effect over a range of values.

The data indicates the effect of L/D in all régimes for a Prandtl number variation of two orders of magnitude. The necessary L/D for effectively infinite length behaviour in natural convection is apparently less than in mixed and in forced convection. However, it is surprisingly large in all régimes and apparently increases with Prandtl number. It is known (see Schorr & Gebhart, 1970) that the distant flow induced in natural convection results in a large in-flow in the third or axial dimension. For a given L/D the finite length effect was found here to be large in forced flow and it increases rapidly with fluid viscosity. This suggests that the axial flow is induced by the blockage which results from the low velocity viscous region around the cylinder. Calculations show that the finite length effect cannot be accounted for through any simple end-loss correction.

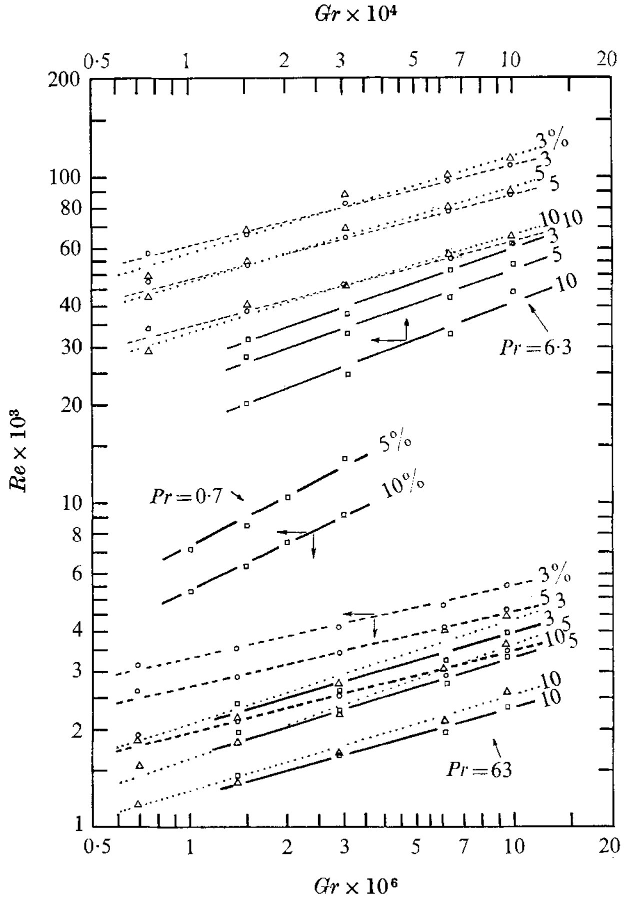

FIGURE 9. Limits of the forced-convection régime. Percentage changes in transport due to natural convection are indicated. L/D: □—□, 8000; ○- - -○, 12,000; △...△, 16,000.

Most applications, such as hot-wire anemometry, involve relatively small L/D values. This is unfortunate since the large remaining finite length effect, even at large L/D, apparently indicates that an analysis must consider very complicated aspects of the flow.

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